Sec 1.4 Separable ODEs

So far, looked at first order (no y'', y''', ...) eq $\frac{dy}{dx} = f(x,y)$

- Eqs $\frac{dy}{dx} = f(x)$ can be solved by just integrating.

- Separable Eq: ODE $\frac{dy}{dx} = f(x,y)$ in which $f(x,y)$ can be separated as a product $f(x,y) = g(x) \cdot h(y)$

- Ex: Find a general soln for $\left\{ \frac{dy}{dx} = -6xy \right\}$

$$\frac{dy}{dx} = (-6x)(y) \longrightarrow \int \frac{dy}{y} \overset{*}{=} \int -6x\,dx$$

* for now, assume $y \neq 0$.

$$\ln|y| = -3x^2 + C$$

$$|y| = e^C \cdot e^{-3x^2}$$

$$y = \underbrace{\boxed{\pm e^C}}_{\text{"}C\text{"}} e^{-3x^2}$$

initially, since we said $y \neq 0$, we have to say $C \neq 0$ too.

a general solution is* $\boxed{y = Ce^{-3x^2} \quad (C \neq 0)}$

But is $y = 0$ actually also a solution?

$\rightarrow$ plug in, $y' = 0$, $y = 0$ $\rightarrow \left\{ \begin{array}{l} y' = -6xy \\ 0 = -6x \cdot 0 \checkmark \end{array} \right\}$

so $y \equiv 0$ ($\equiv$ means "equal everywhere") is a solution,

so our "full" general solution is

$$\boxed{y = Ce^{-3x^2} \quad C \in \mathbb{R}}$$

"in" (b/c "element of")

("C can be any real #")

(explicit general solution)

- "Extra" solution that is a constant (ex: $y \equiv 0$) is an "equilibrium solution"

"Natural Growth / Decay" Eq: $\frac{dy}{dx} = ky$ or $\frac{dx}{dt} = kx$, etc...

★ Read examples 3 & 4 in book.

Ex of Separable Eq: Newton's Law of Cooling

If an ambient temperature of $A = 400°$, const. $k = 3/2$, and an initial temperature of object, $T(0) = 60°$, are given, then "model"* the temperature $T(t)$ over time.

(* find solution of diff eq)

$$\frac{dT}{dt} = k(A - T) \longrightarrow \frac{dT}{dt} = \frac{3}{2}(400 - T)$$

$$\rightarrow \int \frac{dT}{400 - T} = \int \frac{3}{2}\,dt$$

$$-\ln|400 - T| = \frac{3}{2}t + C$$

$$|400 - T| = e^{-C} e^{-3/2\,t}$$

$$400 - T = \overbrace{\boxed{\pm e^{-C}}}^{C} e^{-\frac{3}{2}t}$$

$$\boxed{T = 400 + Ce^{-3/2\,t}}$$

general solution for $\left\{ \frac{dT}{dt} = \frac{3}{2}(400 - T) \right\}$

To find particular solution with $T(0) = 60$, solve for C:

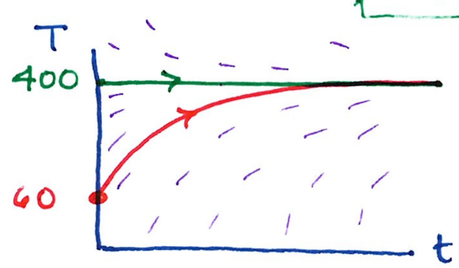
$$60 = 400 + C\overset{1}{\cancel{e^{0}}}$$

$$60 = 400 + C$$

$$C = -340$$

(* $T(t) \equiv 400$ is an equilibrium soln)

particular solution is $\boxed{T(t) = 400 - 340e^{-3/2\,t}}$



Ex : Find an "implicit" general solution for

$$\frac{dy}{dx} = \frac{4-2x}{3y^2-5},$$

then find a particular solution having $y(1) = 3$

implicit soln : $F(x,y) = C$

$$\int (3y^2-5)\,dy = \int (4-2x)\,dx$$

$$y^3 - 5y + C_1 = 4x - x^2 + C_2$$

$$\boxed{y^3 - 5y + x^2 - 4x = C}$$ implicit general solution.

- For particular solution with $y(1) = 3$, remember this means curve(s) pass through $(x=1, y=3)$, plug that in

$$\longrightarrow \quad 3^3 - 5(3) + 1^2 - 4(1) = C$$

$$\cancel{27 =}\ C = 9$$

particular sol : $y^3 - 5y + x^2 - 4x = 9$